Hello

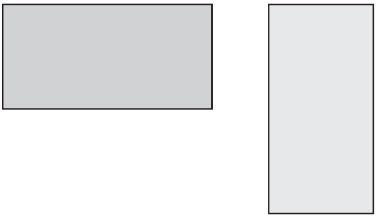
This booklet is for everyone. If you have a few spare minutes, why don't you take a look? You might find it interesting.

If you read this, please take your time and give thought to the questions raised. If you don't find this interesting, please give it to someone you feel would appreciate it.

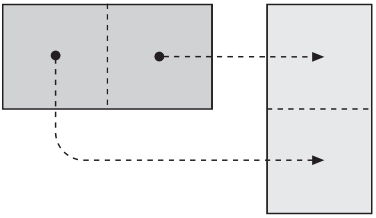
Introduction - What is area?
(or: when do two shapes have the same area?)

Do these two squares have the same area? They are identical in all aspects, so they must also have the same area.

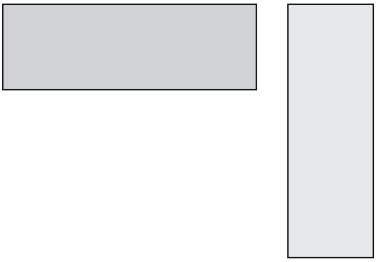
What about these two rectangles? They are not identical. Do they still perhaps have the same area?



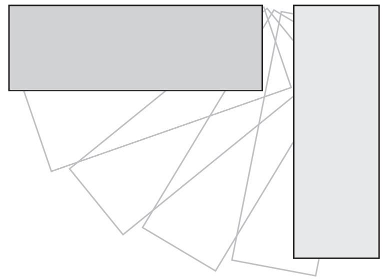
In this case, we can cut up the left rectangle into two identical squares, and “slide” these squares to fit perfectly into the right rectangle. Here we see that the left rectangle and the right rectangle are built up of the same squares, so they also have the same area.



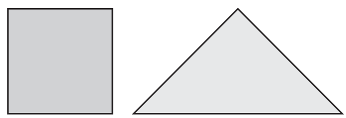
What about these two rectangles? If you try to split the left rectangle into squares, as we just did, you’ll always end up with some area left over.



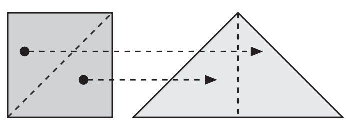
But still, if you rotate the left rectangle it fits perfectly into the right one, and since rotation does not change the area of a shape, these two rectangles also have the same area.



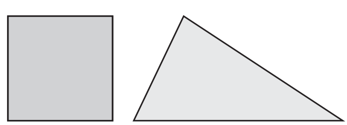
Now let's ask some more interesting questions. Do these two shapes have the same area? This is a bit harder, since these are two different types of shapes altogether.



Still, we can cut up the square into two identical triangles, and slide them to fit perfectly into the right triangle. Again, these shapes have the same area. Notice that we needed to rotate one of the parts of the square before sliding it in.



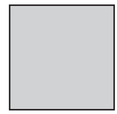
What about these? Can you cut up this square and build this triangle from its parts? At first it may seem impossible. At this point, you might assume that they don't have the same area. But if you put more thought into it, you could possibly find a way.



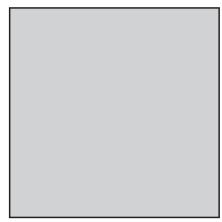
The question arises: When can you know for sure that two shapes **do not** have the same area? This is the main question that will guide our discussion.

Doubling the square (part 1)

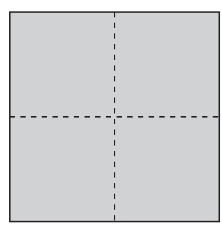
Let's look at some other questions in order to broaden our understanding of area. Here's a square. Can you find a new square with precisely twice its area?



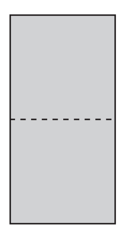
We might try doubling each side of the square, like this one. It might seem logical that this square has twice the area as the original one...



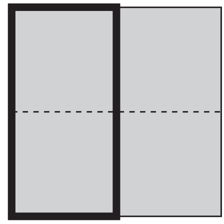
...but we can split this new square into **four** squares, each of which is completely identical to the original square! So, this square actually has **four** times the original square's area, not two like we're looking for.



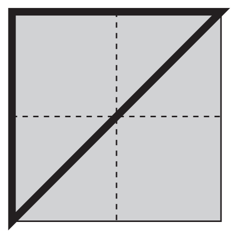
What shape **does** have twice the area of the original square? We can construct a rectangle by sticking two of those squares together. This rectangle has the same area as the square we're looking for. So, can we find a square with the same area as this rectangle?



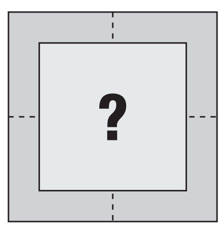
Now, we can look at this rectangle as half the area of the second square we constructed, thus finding yet another way of asking the same question: Can we find a square with **half** the area of this larger square?



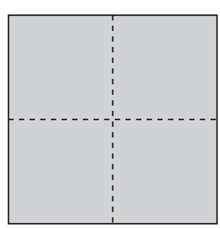
And what ways are there to cut a square in half? We've seen a rectangle as half of a square, and we can also cut the square on its diagonal, reaching this triangle. So this triangle has the same area as the square we're looking for, but, well, it's not a square...



It seems like we haven't gotten any further at all in solving our problem, but actually what we have just done is play around with different directions that might lead to solutions. Although none have, we have come up of different possibilities and the final solution will utilize many of these ideas.

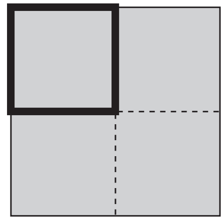


Think about it. We're looking for a **square** having half the area of this square. We've seen a rectangle having half its area and a triangle having half its area. Can you think of a way of halving this large square and still staying with a square?

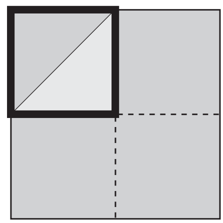


Doubling the square (part 2)
(or: when don't two shapes have the same area?)

We tried halving this larger square, during which we found a triangle and a rectangle, but no square. Let's start by trying to halve only a quarter of this larger square.

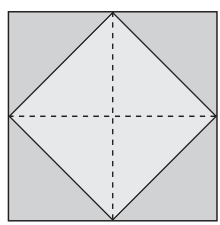


If we halve the small square like this, we find that the lighter area has half the area as the small square. Can we halve the other three squares and find our square?



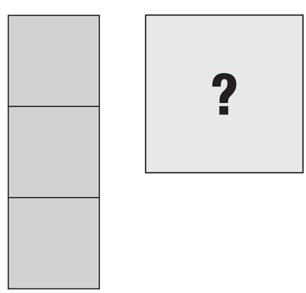
Here it is! In each small square, the light area is half the area of the whole small square. Altogether, the light area has half the size of the large square. This means the light area has precisely the area we were looking for.

And, it's a square! The light square has twice the area of the original square - the one we were trying to double!

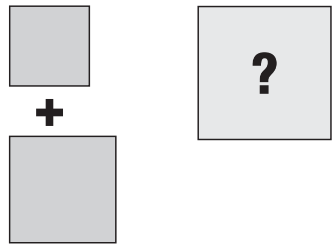


Well, we found an answer to our original question in an interesting way. This leads us to some additional related question, each having to do with the same ideas we used. These questions are intended to be thought of over time.

1. Can you construct a square with the same area as **four** times a given square? How about three or five?

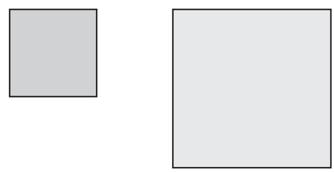


In our first question, we found a square with twice the area of a given square. Twice the area is the same as adding two copies of the same square together. We might want to ask a more general question:

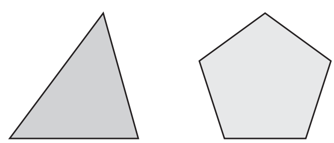


2. Given two (not necessarily equal) squares, can you find a square whose area is the equal to those two squares combined?

Let’s go back to a question we asked earlier. When do two shapes **not have** the same area? We know it’s not always a simple question, but what about these two squares? Their lengths are different, so they must not have the same area. Could this be helpful for comparing areas of shapes that are **not** squares?

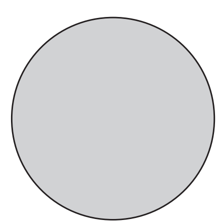


Look at these two shapes. It looks like there is no way we could tell if they have the same area or not. But if we could somehow transform each one of them into a square, we would know immediately if they have the same area. So in fact, we can reduce our original question to:



3. Can any shape with straight lines be transformed into a square?

4. What about a circle? Is there a square with the same area as a circle? How could we find such a square?



Geometry

began in Ancient Egypt, where so-called rope-strechers were hired to measure distances and areas. At that point, geometry had only a practical purpose. Around the 6th century BC, a greek wise man named Pythagoras of Samos travelled to Egypt to study geometry. Pythagoras realized that geometry, astronomy, arithmetic and music are all related. He reached the conclusion that everything can be found in numbers and numerical relations.

When he returned to Greece, he opened a school, where mathematics (originally meaning 'the study') was studied and discussed. Pythagoras realized that mathematics, in addition to its practical purpose, has a **personal** purpose. One who studies mathematics becomes, with time, able to see things he had not been able to see before. His students, later known simply as Pythagoreans, spread the ideas of their school, and from generation to generation, more and more people joined their study.

The later Pythagoreans came to write some of the more influencial mathematical books in history. The most important one of these is 'The Elements' by Euclid, written around 300 BC. This book summarized all of the studies of the mathematicians of Greece, starting from Pythagoras and up to his day. Today, Euclid's Elements is still considered one of the best written mathematical books in existance, and it is surely the most widely distributed one. Yet most probably none of you have ever seen it or even heard of it.

There are many ways to teach mathematics. When mathematics is studied for the sole purpose of ascending your own personal understanding, you can find beauty and purity which you can hardly find anywhere else.

If you found this booklet interesting, you should know that it is just the beginning. We are looking for people who would like to join this type of study.

For more information, go to:
www.theWE.net/math